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Two-sided Markets with Heterogeneity in Users' Interaction Value*

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ABSTRACT

This paper develops a simple two-sided platform model where users are heterogeneous in their interaction value. It is shown that when it is optimal for a monopoly platform to admit all sellers, the size of participants on the buyer's side is larger than that of the social optimum. Conversely, if the platform wishes to exclude some sellers, the size of buyer-side participants is smaller than that of social optimum. If price discrimination is allowed, the platform can draw socially optimal sets of subscribers by eliminating a Spence-type distortion. Social optimum is also achievable under two-part tariffs based on transaction volume.

Key words: Multi-sided platform, User heterogeneity, Price discrimination
JEL Classifications: D42, D85, L12, L14

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I. INTRODUCTION

This paper investigates pricing strategies of a monopoly platform when users on each side are heterogeneous in their interaction benefit created. It develops a tractable two-sided platform model, examines strategies of the platform, and evaluates their welfare effects.

Platform businesses played a central role in promoting innovations and enhancing competition in various markets and industries. Example include media industries, software/application markets, payment markets, intermediation service markets, among others. Owing to their impact on the market structure and performance, the strategies of platforms with the so-called "two-sided market" have been one of the hot topics in the field of industrial organization.

Most studies in this area have considered cross-platform externality, intrinsic to recently introduced platform businesses, as a critical feature of the platform[1]. They found that traditional economic theories are not applicable to two-sided market, and characterized the optimal pricing strategy of monopoly platforms or competing platforms. The central issues are the relative price between groups on different sides, and whether the size of participants on the platform is socially optimal.

This paper also examines the optimal strategy of monopoly platform, but considers a different type of heterogeneous users from previous literature. Most studies on two-sided platforms have assumed that the cross-platform externality a user on one side creates to the other side is identical across users. Under this assumption, the interaction benefits for a user on one side depend on the 'number' of participants on the other side.

To see this formally, consider the simplified model of Weyl (2010) where the benefit of user i on side j is characterized as the following formula:

$$U_i^j = B_i^j + b_i^j N^{-j} - P(\cdot),$$

[1] For definition and characteristics of two-sided markets, see Rochet & Tirol (2006) and Rhee (2010).

Where B_i^j is the membership benefit, and b_i^j is the coefficient of usage/ interaction benefit, N^{-j} is the number of users on the other side, and $P(\cdot)$ is the money transfer from the user to the platform. As Weyl (2010) argues, two canonical two-sided market models −Rochet & Tirole (2003) and Armstrong (2006)− posit the user heterogeneity in different ways. Rochet & Tirole (2003) assumed $B_i^j \equiv 0$ and the heterogeneity of users are imposed on interaction benefit b_i^j. To the contrary, Armstrong (2006) assumed that the interaction value is homogeneous($b_i^j = b_j$), but the membership benefit is heterogeneous across users. In both models, however, it is assumed that cross-platform externality depends on the number of participants on other side of the platform(N^{-j}). That is, all users on the other side create identical interaction value. Weyl (2010) and Rochet & Tirole (2006) presented more general model where both the membership benefit and the interaction benefit are heterogeneous, but they still assume the cross-platform effect of a user on the other side is homogeneous[2].

Unlike previous studies. this paper builds a simple model of two-sided market with heterogeneous users in interaction value, particularly, size network effect across platform. I relax the assumption that users are homogeneous in creating the interaction value. By doing so, it investigates whether the optimal pricing by the monopoly platform departs from the social optimum, and how the platform mitigates such distortion.

The main finding in this paper is that the social optimum is not achieved by a uniform pricing strategy, even when there are no other types of heterogeneity presumed by previous studies. The result is quite intuitive. As the monopoly platform only cares about the marginal user, while ignoring the average user, the monopolist induces the user size that is not social optimal. This type of distortion is already well known in traditional economic theories, noticed first by Spence (1975) and applied to the two-sided market by Weyl (2010). This paper confirms that the theory works in the case of heterogeneous users in interaction value. It also show that the distortion can be eliminated when the platform can discriminate

[2] Rochet & Tirole (2006) are aware of the limitation of the canonical model as they note "implicit assumption is that side i cares, on the other side, only about the number of users N^j" where in their notation i and j denote sides opposite to each other. However, they do not provide a formal analysis for this.

prices among users with a two-part tariff associate with the interaction value, as conventional theories on price discrimination tell us.

One of interesting and noble findings of my analysis is that whether the monopoly pricing is sub-optimal or super-optimal in terms of user size depends on whether the platform excludes some of users on one side. When the platform serves all users on one side(the seller's side in the model), the benefit of the marginal user on the other side(the buyer's side) exceeds the average benefit of the intra- marginal users, which results in super-optimality. Conversely, when the platform excludes some of users on one side, the benefit of the marginal user on the other side is less than the average benefit of the intra-marginal users, which results in sub-optimality.

The results could provide policy implications for anti-trust cases or on competition policy. In most two-sided markets, the potential users are easily identifiable on the seller's side but not so on the buyer's side. Nonetheless, the interaction values are created by the buyer side with heterogeneity. In this case, this study suggests that with full participation of sellers, too big platform might be the problem, while with partial participation of sellers, too small a platform from monopoly pricing might matter.

There are several studies related to the study, apart from theory papers on two-sided markets mentioned earlier. Since the model in this paper is a good fit for intermediation platforms and payment card platforms, several papers on these specific markets, including Cabral (2005), Caillaud & Jullien (2003), Rochet (2007), Rochet & Tirole (2011), Yoon (2018), and so forth, share similar features with this paper. This paper presents a simple but quite general model to clearly show the source of the distortion addressed by previous literature. Theoretical studies, for example Poolsombat & Vernasca (2006), have also considered heterogeneous interaction value, but the heterogeneity in there model is imposed on platform that users participate in not on the users themselves. Recent studies on price discrimination by platforms, such as Liu & Serfes (2013) and Carroni (2015), have taken user heterogeneities into considerations as well. However, they did not include the heterogeneity in interaction values and the focus of their analysis is different from mine.

The rest of the paper is organized as follows. Section II describes the model. In section III, I discuss the socially optimal subscription. Section IV characterizes the optimal strategy of the monopolistic platform, without and with price discrimination, respectively. The final section concludes and proposes topics for future research.

II. MODEL

An asymmetric two-sided platform is modeled to analyze the monopoly platform's pricing strategy. On side S, which I call 'seller's side,' there are two potential users $j \in J = \{1,2\}$. On side B, which I call 'buyer's side', there are continuum of users i distributed on $I = (0,1]$ with density function of $f(i)$. The buyer index i represents total willingness to pay for both seller, and I call it simply 'budget'[3]. is I impose an additional assumption that buyers are uniformly distributed on its support, i.e., $f(i) = 1/\bar{i}$ if $i \in I$ and zero otherwise.

Throughout this paper, I call a user on side S a seller, and a user on side B a buyer. We can think of the users on the seller's side as advertisers on a newspaper platform, merchants on a payment card platform, or application developers on an operating system platform or app store platform. Likewise, the users on the buyer's side can be thought of as readers, payment card holders, users of an operating system, respectively. Note that in the above examples, there are relatively fewer users on the seller's side and many users on the buyer's side, which justifies the asymmetry of the model.

It is assumed that each buyer spends all his budget on purchasing either from seller 1 or from seller 2. Then, letting $x_{j,i}$ be the amount of money for the buyer i to purchase goods from seller j and $x_j \equiv \sum_i x_{j,i}$, we have $x_{1,i} + x_{2,i} = i$. To avoid multi-dimensionality in the buyer's heterogeneity, the proportion of expenditure on each seller is assumed to be independent and identical across buyers.[4] We fix the proportion to be $\mu_j \equiv x_j/(x_1 + x_2)$ and $\mu \equiv \mu_1 > 1/2$ without

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[3] I abuse notations by letting i represent both user i and the budget of user i.

[4] I implicitly assume that buyers' transaction choice is not affected by the platform choice. For instance, an automobile buyer purchases the car he wants even though the car seller does not receive payment cards.

loss of generality. This implies that the seller 1 creates a greater, possibly in the negative direction, volume of cross-platform externality once it joins the platform.

The utility of buyer i is composed of membership benefit and interaction benefit as is in Rochet & Tirole (2006) and Weyl (2010). Let $\hat{J} \subset J$ be a set of sellers who join the platform. Then, the utility of buyer i is specified as follows[5]:

$$U_i = u_i + \sum_{j \in \hat{J}} \beta_i \mu_j i - P_b(\cdot) \tag{1}$$

In the utility function, u_i represents the membership benefit, which is assumed to be identical across buyers ($u_i = u$, for all $i \in I$). The membership benefit can be thought of as the reduction of search and transaction costs. The next term $\sum_{j \in \hat{J}} \beta_i \mu_j i$ represents interaction benefit. The benefit is created only when the counter-party of the transaction joins the platform, and the volume of the benefit is assumed to be linear in the transaction size. For a detailed explanation of interaction benefit, see Appendix A.1[6]. I also assume that $\beta_i = \beta$, for all $i \in I$. The term P_b, the price, is the transfer from the buyer to the platform, and is set by the platform. Without price discrimination, the platform charges uniform price across buyers. With price discrimination, the platform could condition P_b on each buyer's type, the budget i, as long as the platform can identify it.[7] We allow negative pricing.

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[5] In the utility function, each user exhausts his entire budget to buy goods from seller 1 and seller 2, but still has to pay the fee charged by the platform(P_b), which looks odd. In this sense, the term 'budget' would be misleading. The utility specification can be justified by the fact that on the platform users can get benefits from reduced transaction/search costs(u_i) and successful matching($\beta_i x_{i,j}$) which they cannot get off-the-platform. These benefits enable buyers to afford the platform fee. Thus, the budget is 'soft.' Despite the risk of misleading, I use the term 'budget' for simplicity. I appreciate an anonymous referee for pointing this out.

[6] The model presented here is a version of `macro' model in terms of Economides (1996). Appendix A.1. provides a partial `micro' model.

[7] In reality, the price can be charged proportional to the transaction volume. In the model here, however, each buyer spends fixed amount determined ex ante. Therefore whether the price is charged based on the transaction volume or on the type(budget) of buyer does not matter.

The sign of the membership benefit and interaction benefit depends on the characteristics of the platform. For example, in newspaper, the membership benefit is strictly positive while the interaction benefit is negative. In case of intermediary platform such as payment cards and app stores, the membership benefit is almost zero, while the interaction benefit is strictly positive. While the model is quite general, to avoid too many cases being considered, I will examine only the case of positive interaction value in the analysis. Along with the assumption that the interaction benefit of users on the other side is linearly correlated, the model fits well for intermediation platforms and payment card platforms with a fixed margin rate.

As for the seller's side, it is again assumed that the interaction benefit is proportional to the volume of transaction through the platform. Letting $\hat{I} \subset I$ be the set of buyers subscribing to the platform, the seller's utility(profit) from the platform is specified as

$$V_j = v_j + \int_{i \in \hat{I}} \gamma_j x_{i,j} f(i) di - P_s(\cdot) = v_j + \int_{i \in \hat{I}} \gamma_j \mu_i i f(i) di - P_s(\cdot) \qquad (2)$$

, where v_j is the membership benefit and P_s is the transfer(price) from seller j to the platform. Membership benefit can be though of as a reduction in advertising and consumer service costs. The interaction benefit is assumed to be linear in the transaction size, where γ represents the ratio of the benefit of the transaction volume. It could represent the fixed margin rate in cases of intermediation services or payment card services. Without price discrimination, P_s is the same across sellers. With price discrimination, the platform can charge different prices to different sellers. It is assumed that $v_1 = v_2 \equiv v$ and $\gamma_1 = \gamma_2 \equiv \gamma$.

To provide service, the platform incurs a cost on the buyer's membership(c_s), on the seller's membership(c_b), and on interactions(c_x). It is assumed that the membership cost is linear in the number of members, while the interaction cost is linear in the number of interactions. For notational convenience, define the function of member size $|\cdot|$. For side $\boldsymbol{S}$, $|\hat{J}|$ is just the cardinality of $\hat{J}$, and for side $\boldsymbol{B}$, $|\hat{I}| \equiv \int_{i \in \hat{I}} f(i) di$. The cost function is then

$$C(\hat{J},\hat{I}) = c_s \sum_{j\in \hat{J}} 1(j) + c_b \int_{i\in \hat{I}} f(j)di + c_x \sum_{j\in \hat{J}} \int_{i\in \hat{I}} \mu_j i f(i) di \\ = c_s|\hat{J}| + c_b|\hat{I}| + \sum_{j\in \hat{J}} \mu_j E[i|j] \tag{3}$$

, where $1(\cdot)$ is an indicator function and $E[i|j]$ is the conditional expectation of buyer's budget i given j.

The profit of the platform at price $\boldsymbol{P} \equiv (P_s, P_b)$ and resulting sets of users $(\hat{J},\hat{I})$ is then

$$\Pi(\hat{J}(\boldsymbol{P}),\hat{I}(\boldsymbol{P}),\boldsymbol{P}) = \sum_{j\in \hat{J}} (P_s(j) - c_s) + \int_{i\in \hat{I}} (P_{b(i)} - c_b) f(i) di \\ + c_x \sum_{j\in \hat{J}} \int_{i\in \hat{I}} \mu_j i f(i) di. \tag{4}$$

As noted above, I will focus on the positive interaction benefit of the platform. In particular, total interaction benefit, the sum of benefits of buyers and sellers is assumed to be strictly greater than the relevant cost. Further, it is assumed that sellers' membership benefit is the same as the cost for the membership, which is plausible in most of the intermediation platforms where interaction benefits are more important.

Assumption 1

1. *The total interaction benefit is greater than its relevant cost, i.e.,* $\beta + \gamma > c_x$.
2. *Sellers' membership benefit is the same as its relevant cost, i.e.,* $v = c_s$.

The timing of the game is as follows. First, the platform presents the prices P_s and P_b. Then, the sellers and buyers simultaneously decide whether to join the platform. Finally, transactions take place and benefits are realized and enjoyed. The Subgame Perfect Nash Equilibrium is adopted as a solution concept.

III. SOCIALLY OPTIMAL ALLOCATION

Before I analyze the monopoly pricing, I first investigate the socially efficient allocation. Social surplus(SW) depends on the set of members in both sides $(\hat{J}, \hat{I})$, which is given by

$$\begin{aligned} SW(\hat{J}, \hat{I}) &= \sum_{j \in \hat{J}} \int_{i \in \hat{I}} \gamma \mu_j i f(i) di \\ &+ \int_{i \in \hat{I}} \left(u - c_b + \sum_{j \in \hat{J}} \beta \mu_j i \right) f(i) di \\ &- c_x \sum_{j \in \hat{J}} \int_{i \in \hat{I}} \mu_j i f(i) di \\ &= (v - c_s)|\hat{J}| + (u - c_b)|\hat{I}| + (\beta + \gamma - c_x) \sum_{j \in \hat{J}} \mu_j E[i|j] \end{aligned} \tag{5}$$

Solving the maximization of the social surplus function, the following proposition is obtained.

Proposition 1 *Suppose* ***Assumption 1*** *holds. Then,*

1. *If the membership benefit for buyers is greater than its cost ($u > c_b$), then in a socially optimal allocation all buyers and both sellers join the platform: $\hat{I} = I$ and $\hat{J} = \{1,2\}$.*
2. *If the membership benefit for buyers is less than its cost ($u < c_b$), in the social optimum, only some of the buyers join the platform, and the seller 2 may not participate in the platform. Specifically, the optimal set of buyer members is $[\hat{i}, \bar{i}]$, where*

$$\hat{i} = \begin{cases} \dfrac{c_b - u}{\mu(\beta + \gamma - c_x)} & \text{if } \hat{J} = \{1\} \\ \dfrac{c_b - u}{\beta + \gamma - c_x} & \text{if } \hat{J} = \{1,2\} \end{cases} \tag{6}$$

3. *If the membership cost for buyers is high enough compared to its membership benefit ($u \ll c_b$), and/or the interaction benefit is low enough ($\beta + \gamma \approx c_x$), then only seller $j = 1$ joins the platform in the socially optimal allocation.*

Proof. See Appendix A.2.

The results of Proposition 1 are quite intuitive. If both membership and interaction benefit are greater than its corresponding costs, it is socially optimal for both buyers and sellers to join the platform. If, however, the membership benefit is lower than its cost, it is optimal for buyers to join the platform the platform only when the loss from the membership is recompensed by the interaction benefit, in which case the compensation is easier if more sellers join the platform. Finally, if the membership cost for a buyer is very high and/or the interaction benefit is low, then it costs too much to admit more buyers, which in turn makes it optimal to exclude one seller.

IV. MONOPOLY PRICING

1. Uniform Pricing

Now I investigate the pricing strategy of the monopoly platform. I begin with the buyer's choice. Buyer i subscribes to the platform if and only if

$$u + \sum_{j \in \hat{J}} \beta \mu_j i \geq P_b. \tag{7}$$

Let $\tilde{i}$ be the threshold buyer who is indifferent between subscription and non-subscription. Then, we have

$$\hat{i} = \begin{cases} 0 & \text{if } P_b \leq u \\ \dfrac{P_b - u}{\beta \sum_{j \in \hat{J}} \mu_j} & \text{if } u < P_b \leq u + \beta \bar{i} \sum_{j \in \hat{J}} \mu_j \\ \bar{i} & \text{if } P_b > u + \beta \bar{i} \sum_{j \in \hat{J}} \mu_j \end{cases} \tag{8}$$

To avoid uninteresting cases, I examine only the case of interior solutions where only some of the buyers join the platform. On the seller's side, seller j subscribes to the platform if and only if

$$v+\int_{i\in \hat{I}}\gamma\mu_j i f(i)di = v+\gamma\mu_j E[i|\hat{J}\,] \geq P_s \tag{9}$$

I consider the following two cases depending on the set of subscribed sellers.

Case I: Platform serves only one seller.

In this case, only the seller $j=1$ subscribes to the platform. Given the price P_b, the threshold buyer is $\tilde{i}_1=(P_b-u)/(\mu\beta)$. Further, for seller $j=1$ to subscribe to the platform, the inequality $\gamma\mu E[i|[\tilde{i}_1,\bar{i}\,]] \geq P_s$ should be satisfied, which is a binding constraint to maximize the profit. Now, the profit function is modified to the function of $\tilde{i}_1$ alone as follows.

$$\begin{aligned} \Pi &= (P_s-c_s)E[P_b-c_b|[\tilde{i}_1,\bar{i}\,]]-c_x\mu E[i|[\tilde{i}_1,\bar{i}\,]] \\ &= (v+\gamma\mu E[i|[\tilde{i}_1,\bar{i}\,]]-c_s)+E[\mu\beta\tilde{i}_1+u-c_b|[\tilde{i}_1,\bar{i}\,]]-c_x\mu E[i|[\tilde{i}_1,\bar{i}\,]] \\ &= \mu(\gamma+\beta-c_x)E[i|[\tilde{i}_1,\bar{i}\,]]+E[u-c_b|[\tilde{i}_1,\bar{i}\,]]+\mu\beta[\tilde{i}_1-i|[\tilde{i}_1,\bar{i}\,]] \\ &= SW(\{1\},[\tilde{i}_1,\bar{i}\,])-\mu\beta E[i-\tilde{i}_1|[\tilde{i}_1,\bar{i}\,]] \end{aligned} \tag{10}$$

The second term of the last line in (10) is the average interaction benefit of the intra-marginal buyers given a fixed seller. Note that

$$\frac{dE[i-\tilde{i}_1|[\tilde{i}_1,\bar{i}\,]]}{d\tilde{i}_1}=-\int_{\tilde{i}_1}^{\bar{i}} f(i)di$$

, which is increasing in $\tilde{i}_1$ and positive on the support of i. With the equation (a2) in Appendix A.2, differentiation of the profit function is now

$$\frac{d\Pi}{d\tilde{i}_1}=-(u-c_b)f(\tilde{i}_1)-(\beta+\gamma-c_x)\mu\tilde{i}_1 \tag{11}$$

From the first order condition, we get the optimal set of buyers $[\widetilde{i}_1^*, \overline{i}\,]$ for the given set of sellers, which is smaller than what we get from the social optimum. The result shows that the platform under-provides its service, of which possibility is noted by Cabral (2005).

Case II: Platform serves both sellers.

Now, let $\hat{J} = \{1,2\}$. Similarly to the case of one seller, given the price P_b, the threshold buyer is $\widetilde{i_2} = (P_b - u)/\beta$. Further, for seller $j = 2$, as well as $j = 1$, to subscribe to the platform, it must be that $\gamma(1-\mu)E[i|[\widetilde{i_2}, \overline{i}\,]] \geq P_b$, which is binding. The profit function is then given by

$$\begin{aligned} \Pi &= 2(P_s - c_s) + E[P_b - c_b|[\hat{i_2}, \overline{i}\,]] - c_x E[i|[\hat{i_2}, \overline{i}\,]] \\ &= (\beta + \gamma - c_x)E[i|[\hat{i_2}, \overline{i}\,]] + E[u - c_b|[\hat{i_2}, \overline{i}\,]] + \gamma(1-2\mu)E[i|[\hat{i_2}, \overline{i}\,]] \\ &= SW(\{1,2\}, [\hat{i_2}, \overline{i}\,]) - \gamma(2\mu - 1)E[i|[\hat{i_2}, \overline{i}\,]] \end{aligned} \tag{12}$$

Again, given a fixed $\hat{I}$, the profit is smaller than the social surplus. Interestingly, however, the source of distortion is different from Case I. The second term of (12) denotes the difference in the interaction benefit of the two sellers, which implies, along with the assumption of $v = c_s$, that the platform has to compensate for the interaction benefit to include *an additional member* on the other side. This constitutes a Spence-type distortion noted by Weyl (2010). Differentiating the profit function, the following is obtained:

$$\frac{d\Pi}{d\hat{i_2}} = -(u - c_b)f(\hat{i_2}) - (\beta + 2\mu\gamma - c_x)\hat{i_2}f(\hat{i_2}) \tag{13}$$

To compare with the social surplus maximization problem in Appendix A.2, (13) has the same constant but a steeper slope than (a5). This implies that, from the first-oder condition, the set of buyers subscribing to the platform is larger than the social optimum. The result confirms the results of Weyl (2010) and Rochet & Tirole (2011) in the context of the payment card platform. The analysis in this subsection is summarized in the following proposition.

Proposition 2 *Suppose **Assumption 1** holds. Then, the optimal pricing of the platform monopolist without price discrimination satisfies the following properties:*

1. *If the platform wishes to include only one seller, then the set of buyers subscribing to the platform is smaller than that of the social optimum:* $\widehat{i^{SO}} < \widehat{i_1^*}$.
2. *If the platform include both sellers, the set of buyers subscribing to the platform is larger than that of the social optimum:* $\widehat{i^{SO}} < \widehat{i_2^*}$. *In this case, with interior solution,*

$$\widehat{i_2^*} = \frac{c_b - u}{\beta + 2\mu\gamma - c_x} \tag{14}$$

$$P_b^* = u + \beta \widehat{i_2^*} \tag{15}$$

$$P_s^* = v + \gamma(1-\mu)E[i|[\widehat{i_2^*}, \bar{i}\,]] \ . \tag{16}$$

2. Pricing Conditional on Interaction Value

I now consider the case where the monopoly platform can make the pricing conditional the pricing on the interaction value. That is, the platform can implement the price discrimination. I assume the platform identifies each buyer's budget and the seller's type. Thus, the price is now $P_b(i)$ and $P_s(j)$. I focus on the case where the optimality is achieved when both sellers subscribe to the platform. In this case, buyer i subscribes to the platform if and only if $u + \sum_{j \in \hat{J}} \beta\mu_j i \geq P_b(i)$, which is binding for all buyers. Let $\hat{i_3}$ be the threshold buyer. Then, seller j subscribes to the platform if and only if

$$v + \int_{i \in \hat{I}} \gamma\mu_i i f(i) d_i = v + \gamma\mu_j E[i|[\hat{i_3}, \bar{i}\,]] \geq P_s(j)$$

, which is binding as well. The profit function is then,

$$\begin{aligned}\Pi^{PD} &= \sum_{j\in\hat{J}}(P_s(j)-c_s)+E[P_b(i)-c_b|[\widehat{i_{3,}}\bar{i}]]-c_xE[i|[\widehat{i_{3,}}\bar{i}]] \\ &= (\beta+\gamma-c_x)E[i|[\widehat{i_{3,}}\bar{i}]]+E[u-c_b|[\widehat{i_{3,}}\bar{i}]] \\ &= SW(\{1,2\},[\widehat{i_{3,}}\bar{i}]).\end{aligned} \tag{17}$$

When the platform is allowed to make the price conditional on user's type, the profit of the platform is equal to the social surplus and thus, the optimal allocation of the platform coincides with the social optimum, confirming the conventional results of the perfect price discrimination. This is due to the elimination of the Spence-type distortion that occurs when the platform cannot discriminate prices.

Another pricing strategy that the platform could adopt is pricing based on the transaction volume. Suppose that the platform charges $p(j)$ per transaction through the platform, in addition to the subscription fee of P_s. That is, the pricing formula for seller j is $P_s(j)=P_s+p(j)$. Then, the profit becomes

$$\begin{aligned}\Pi^{ptf} = &\sum_{j\in\hat{J}}(P_s-c_s)+E[P_b(i)-c_b|[\widehat{i_{3,}}\bar{i}]] \\ &+\sum_{j\in\hat{J}}E[(p(j)-c_x)\mu_j i|[\widehat{i_{3,}}\bar{i}]]\end{aligned} \tag{16}$$

Each seller will subscribe to the platform if and only if $v+E[(\gamma\mu i-p(j)\mu_j i|\hat{I}]$ is non-negative. Let $P_s=v$ and $p(j)=\gamma$. Then, with the participation constraint of the buyer, we have

$$\begin{aligned}\Pi^{ptf} &= \sum_{j\in\hat{J}}(v-c_s)+E[\beta i+u-c_b|[\hat{i},\bar{i}]]+\sum_{j\in\hat{J}}E[(\gamma-c_x)\mu_j i|[\hat{i},\bar{i}]] \\ &= (\beta+\gamma-c_x)E[i|[\hat{i},\bar{i}]]+E[\beta i+u-c_b|[\hat{i},\bar{i}]] \\ &= \Pi^{PD}\end{aligned} \tag{17}$$

Thus, when the platform charges sellers a price based on the transaction volume in addition to the subscription fee, it can maximize its profit without resorting to price discrimination. The result is summarized in the following Proposition.

Proposition 3

1. *When the platform monopolist can price discriminate, the socially optimal allocation is recouped with the following optimal prices:*

$$P_b^*(i) = u + \beta i \tag{18}$$

$$P_s^*(j) = v + \gamma \mu_j E[i|[\hat{i}^*, \bar{i}]] \tag{19}$$

, where

$$\hat{i}^* = \frac{c_b - u}{\beta + \gamma - c_x} \tag{20}$$

2. *The platform can maximize its profit by charging subscription fee of v and transaction fee $\gamma \cdot x_{i,j}$.*

The results in Proposition 3 are not far from the conventional knowledge that the monopolist can recoup the socially optimal allocation with perfect price discrimination, either through bundling(Proposition 3-1) or a two-part tariff (Proposition 3-2). The interesting feature of these results in the context of two-sided market is that the heterogeneity in the model originates with the buyer, and when the platform wants to price discriminate seller, it should make the price conditional on the function of type of buyers. In this sense, the pricing strategy is like “insulating tariff” or “penetrating price” as termed by Weyl (2010) and White & Weyl (2016). In their analysis, it is assumed that the price for the user on one side can be conditioned on the size of participants on the other side. With heterogeneous but linearly correlated interaction value in my model, the insulating tariff on sellers should be conditioned either on the average interaction benefit of the buyers or on the transaction value/volume. In this respect, the paper is an extension of Weyl (2010) to a model with rich heterogeneity.

Another implication of the results is that when the platform uses a two-part tariff, it does not need to identify sellers type, as the transaction fee does not depend on μ_j. This is what we sometimes observe in real world pricing of platforms. For example, credit card platform charges a fixed subscription fee for card-holders and a two-part tariff based on transaction volume for merchants, possibly, with identical rates.

V. DISCUSSION AND CONCLUSION

This paper analyzes a two-sided platform model when the benefit to users on each side depends on the volume of usage/transaction, as a result allowing heterogeneity in interaction benefit. In the model presented, there are two sellers on one side and continuum of buyers on other side. I show that when the monopolistic platform wants to exclude some of the sellers, the size of participants on buyer's side is smaller than that of the social optimum. Conversely, when it is optimal for the monopolist to admit all sellers, the size of participants on the buyer's side is larger than the social optimum. In the second case, it is shown that the distortion is eliminated by price discrimination or charging a two-part tariff based on transaction volume.

These results could explain why intermediation service platforms such as payment cards or app stores adopt a strategy charging per transaction proportional to the transaction volume on one side, subscription based pricing on the other side. Further, applying a differential rate per transaction would not increase profits if the per transaction benefit for users are homogeneous. The observed differences in the rate could be the result of other factors such as bargaining power.

The results also provide some policy implications. In most two-sided markets, the potential users are easily identifiable on one side but not on the other side. For example, in intermediation services, the potential sellers are easily identifiable but it is difficult to identify the potential buyers. The interaction values are created on the buyer's side with heterogeneity. In this situation, the policy makers should design a measure based on the observations on one side. This study suggests that with full participations of sellers, too big a platform may be the problem, whereas with partial participations of sellers, too small a platform resulting from monopoly pricing could matter. The reasoning could be applicable for anti-trust cases or competition policies.

There are many issues on the pricing strategies by multi-sided platform that the paper does not address. First, the model assumes that the interaction benefit of users are linearly correlated, as it is a proportion of interaction value. The assumption works for some platform businesses but not for others. It would be worthwhile to analyze more general model with different interaction benefits for users on different sides, still maintaining the assumption that the heterogeneity of users on one side affect the interaction benefit of users on the other side. Second, when the platform price discriminates, it is assumed that the transaction volume is deterministic and the platform can identify all types of users, which may not plausible in many situation. Extending the model in this perspectives could lead to an analysis of second degree price discrimination, which would be interesting. Finally, important questions would be addressed when platform competition is incorporated. I leave these research questions for future research.

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< APPENDIX >

A.1. Background Model of the User Heterogeneity

To derive the utility of a buyer from subscribing the platform, several assumptions on per transaction(usage) are imposed. The size of each transaction is normalized to one. Let $c_{k,i} \in \{1,2\}$ be the transaction decision by buyer i at $k \in [0,i]$, specifying with whom he transacts. The benefit per transaction depends on whether the seller joins the platform, in which case he enjoys benefit of i. Note that the source of the benefit is from the platform, not from the seller, and it is measured over the benefit from outside option, e.g. transaction outside the platform. Recall $\hat{J}$ is the set of sellers joining the platform. Let $1(j)$ be the indicator function, the value of which is 1 if $j \in \hat{J}$ and 0 otherwise. The benefit of buyer i per transaction, when she transacts with seller j is $w_i(c_{k,i})\beta_i$. The interaction benefit is now

$$\sum_{j \in \hat{J}} \int_0^i 1(c_{k,i}) w_i(c_{k,i}) dk = \beta_i \sum_{j=1}^{2} i(j) \mu_j i$$

The overall utility is derived by taking the membership benefit of u_i and price for joining the platform P_b into consideration.

A.2. Proof of Proposition 1

Since the benefit increases in i, the set of buyers joining the platform can be rewritten as $\hat{I} = [\hat{i}, \bar{i}]$. The socially optimal allocation could include only one seller or both sellers, and in the former case, seller $j = 1$ would be included because it enhances more interaction benefit. Two cases are examined turn by turn.

Case 1: $\widehat{J^{SO}} = \{1\}$

In case where only one seller is included, the benevolent social planner solves the following problem:

$$\max_{\hat{i}} SW(\{1\}, \hat{I}\,) = (u - c_b)\int_{\hat{i}}^{\bar{i}} f(i)di + (\beta + \gamma - c_x)\mu \int_{\hat{i}}^{\bar{i}} if(i)di \qquad \text{(a1)}$$

Differentiating the social surplus function with respect to $\hat{i}$, we get

$$\frac{dSW}{d\hat{i}} = -(u - c_b)f(\hat{i}) - (\beta + \gamma - c_x)\mu \hat{i} f(\hat{i}) \qquad \text{(a2)}$$

From Assumption 1, the net interaction benefit (interaction benefit less its cost) is positive.[8] In this case, if $u \geq c_b$, i.e., the membership benefit is greater than its cost, the allocation that all buyers join the platform is socially optimal. If the membership benefit is far less than its cost ($u - c_b \leq \bar{i}\,\mu(\beta + \gamma - c_x)$), then the allocation that no buyer joins is socially optimal. If the membership benefit is in the middle, the threshold would be

$$\widehat{i^1} = \frac{c_b - u}{\mu(\beta + \gamma - c_x)} \qquad \text{(a3)}$$

Case 2: $\widehat{J^{SO}} = \{1,2\}$.

When both buyers join the platform, the social surplus is now

$$\max_{\hat{i}} SW(\{1,2\}, \hat{I}\,) = (u - c_b)\int_{\hat{i}}^{\bar{i}} f(i)di + (\beta + \gamma - c_x)\int_{\hat{i}}^{\bar{i}} if(i)di \qquad \text{(a4)}$$

Differentiating the profit function, we get

$$\frac{dSW}{d\hat{i}} = -(u - c_b)f(\hat{i}) - (\beta + \gamma - c_x)\hat{i} f(\hat{i}) \qquad \text{(a5)}$$

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8) If the net interaction benefit is negative, for example in case of newspaper platform, the social surplus function is convex in the threshold $\hat{i}$, which implies that either no buyer member or all buyer member is socially optimal.

Again, in the socially optimal allocation, if $u \geq c_b$, all buyers would join the platform, and if $u - c_b < \bar{i}(\beta + \gamma - c_x)$, then no buyer would join. In the middle value, the threshold buyer satisfies

$$\hat{i^2} = \frac{c_b - u}{\beta + \gamma - c_x} \tag{a6}$$

Comparison

There are two cases we need to consider. First, suppose $u \geq c_b$. Then, whatever the set of joining sellers($\hat{J}$) is, all buyers join the platform. In this case, the social surplus is greater in case of $\hat{J} = \{1,2\}$ than in case of $\hat{J} = \{1\}$, because

$$\begin{aligned} SW(\{1,2\}, I) - SW(\{1\}, I) &= ((u - c_b)|I| + (\beta + \gamma - c_x)E[I]) \\ &\quad - ((u - c_b)|I| + (\beta + \gamma - c_x)\mu E[I]) \\ &= (\beta + \gamma - c_x)(1 - \mu)E[I] > 0 \end{aligned} \tag{a7}$$

Now consider the case of $u < c_b$, where only part of the buyers join the platform. Note $\hat{i^2} = \mu \hat{i^1}$ and thus $\hat{i^2} < \hat{i^1}$, which implies that the set of buyers joining the platform is larger when more sellers join the platform. Then, we have

$$\begin{aligned} &SW(\{1,2\}, [\hat{i^2}, \bar{i}]) - SW(\{1\}, [\hat{i^1}, \bar{i}]) \\ &= \left((u - c_b) \int_{\hat{i^2}}^{\bar{i}} f(i)di - (\beta + \gamma - c_x) \int_{\hat{i^2}}^{\bar{i}} if(i)di \right) \\ &- \left((u - c_b) \int_{\hat{i^1}}^{\bar{i}} f(i)di - \mu(\beta + \gamma - c_x) \int_{\hat{i^1}}^{\bar{i}} if(i)di \right) \\ &= (u - c_b) \int_{\hat{i^2}}^{\hat{i^1}} f(i)di + (\beta + \gamma - c_x) \left(\int_{\hat{i^2}}^{\bar{i}} if(i)di - \mu \int_{\hat{i^2}}^{\bar{i}} if(i)di \right) \end{aligned} \tag{a8}$$

The sign of equation (a8) in indeterminate, but it is easy to see that if the loss from membership benefit of buyers is not high and the interaction benefit is sufficiently high, it is optimal for the monopolist to admit all buyers.